

The History of the Mathematics of Card Counting

Peter Woerde

Introduction

It has been mathematically proven that card counting is a way to beat the casino in the game of 21, also known as black jack. To get to the subject of card counting, some of the basic rules will have to be known by the reader.

Therefore, a short explanation of the rules that are used in some examples are given first. For the explanation of all the main rules of the game, you can read Appendix A. These rules will be the same as explained by Edward O. Thorp in his book *Beat the Dealer*. Although some of these rules are not applicable anymore, we will use these rules for reasons of simplicity.

After we have taken a look at the main rules of the game of black jack, we will take a look at what exactly is basic strategy and why it is needed to get to the subject of card counting. Then I will explain what card counting consists of and how the card counting history developed itself.

Appendix C gives a short outline of how basic strategy is calculated.

The Rules of Black Jack

It is assumed that the reader knows the basic rules of the game of black jack. For those who do not know these rules, a short explanation is given in Appendix A. These rules are the rules as published in "*Beat the Dealer*" by Edward O. Thorp, since it's this book that made counting popular. Therefore, the rules will slightly differ from the rules that are used these days. Later, we will see that these rule changes are a direct result of card counting.

The two rules that are being used in a mathematical example are given below.

Naturals

If the first two cards dealt either to the player or to the dealer consist of an ace and a ten-value card, they constitute what we shall call a "natural" or "blackjack". If a player has a natural and the dealer does not, the player receives 1.5 times his original bet from the dealer. If a player does not have a natural and the dealer does, the player loses his original bet. If both dealer and player have naturals, no money changes hands.

Insurance

If the dealer's up card is an ace, an additional wager is allowed before the draw. A player may put up an additional side bet equal to half his original bet. After the player has decided whether or not to do this, the dealer checks his hole card. If the dealer has a natural, the side bet wins twice its amount. If the dealer does not have a natural, the side bet is lost. The original bet is settled in the usual way, regardless of the side bet.

Basic Strategy

Basic strategy is a close approximation for the best possible way to play, given a set of rules. For example, it is pretty obvious that you should stand when you have a total of 21. If you draw, you will bust and therefore lose your bet.

However, a lot of those decisions are not at all obvious. For example, should you draw a card to a total of 16 against a dealer's ten? Only aces and two's to fives can give you a better hand, so if you hit, you will lose more often than you will win. However, if you stand, you will lose unless the dealer busts. It turns out that this is even worse for the player. Basic strategy now, is a set of rules which tells you exactly when to stand, when to hit, when to split and when to double down so it will cost you the least amount of money in the long run.

Actually, there are two different basic strategies for every set of rules: total dependent and composition dependent. Total dependent means that the strategy takes only the players total and the dealers up card into consideration. Composition dependent means that also the exact cards that form the players total are taken into consideration.

For example, total dependent strategy tells us to draw a card if we have a total of sixteen against a dealer's 10. Composition dependent strategy tells us to stand if our sixteen is made up out of four fours. As our basic strategy, we will use total dependent, since it is much easier to remember and since card counting more or less takes into consideration the composition of the player's hand.

Of course, you can imagine that the basic strategy will change if the rules that are used are different. In Appendix B you can find the basic strategy for the set of rules that is used in Holland Casinos, the only legal casinos in the Netherlands.

Card Counting

Card counting means that you remember the cards that have been played and are therefore not in play anymore. Once you know this, you can find out which cards are still in the deck and from this you can calculate whether you have an advantage over the house or not.

Furthermore, you can adjust your basic strategy if you take into account the cards that are not in the deck anymore. Let's take a look at a very easy example.

Suppose that after a couple of rounds there are only four cards left in the deck: three ten-value cards and one ace. If you play head-on with the dealer, than two things can happen. Either you get a natural and the dealer has 20 or the dealer gets a natural and you receive 20. These events have an equal chance of occurring. Thus, half of the time, you will win 1.5 times your initial bet. The other half of the time you will have a total of 20 and the dealer will have a natural. So, half of the time, you lose your initial bet. This means that for every two times this happens to you, you will win half your bet on average, i.e. you will have an advantage of 25 percent.

Actually, your advantage will be even greater: half of the time the dealer has a natural, he will have an ace as his up card. This gives you the possibility of insurance. You know the dealer has a ten-value card as his hole card, so the dealer has a natural and if you take insurance, you will win this side bet. You will still lose your original bet, but because insurance pays two to one, you don't lose any money. So for every four times this happens, on average you will win 1.5 times your original bet two times, you will lose one time and one time nothing happens. This means you actually have an advantage of 50 percent.

This example shows that strategy changes will help you win if you count cards (basic strategy says never to take insurance). Also, increasing your bet size whenever you have the advantage over the house will make you a lot of money. In fact, varying bet size is even more important than changing your strategy.

How It All Started

In 1956, the Journal of the American Statistical Association published an article called "The Optimum Strategy in Black Jack". It was written by four army men: Roger Baldwin, Wilbert Cantey, Herbert Maisel, James McDermott. In this article, some very interesting information could be found.

First of all, the derivation of the first basic strategy and this basic strategy itself were in the article. Furthermore, the expectation and the conditional expectation were calculated. The conditional expectation is the expectation given the dealer's up card. All these calculations took the four men three years to calculate, since they only had desk calculators to use. The over all expectation found by the four men was -0.6% . This showed that black jack, if played following the rules of basic strategy, was less favourable to the casinos than any other game. For example, the best bet in craps gives the house a 1.4% edge, roulette gives the house a 2.7% or even a 5.4% edge (depends whether you're playing American or French roulette) and most slot machines are programmed in such a way that they will only return about 93% of the money poured into them, which means that the house has a 7% edge on the slot machines.

So this was already a very nice discovery. However, the most important thing for us in the paper, was the suggestion of card counting.

The following sentence made clear, that an even better strategy could be made if the player saw cards that were already out of the deck: "*The optimum strategy was developed under the assumption that the player does not have the time or inclination to utilize the information available in the hands of the players preceding him in the draw.*"

Another sentence however, suggests that this information cannot be used in a scientific manner, but only in an intuitive manner. How wrong they were.

How It Continued

Edward Thorp, professor at the University of California, was about to go on a holiday to Las Vegas with his wife. Just before this, the article named before was given to him by another professor. Since the tiny over-all edge of 0.62% percent, he wrote the strategy on a little card and took it with him on his trip. He purchased ten silver dollars and went to a black jack table and played according to the rules he had written down. Every time Thorp made a decision that was on his little chart, but that was often made the other way around, he would get comments on his play by other players who seemed to know what they did. After a while he was behind eight and one half silver dollars and decided to stop. But the atmosphere of ignorance and superstition that pervaded his little experience securely planted in his mind the suggestion that even "good" players did not know the fundamentals of this game. There might be a way to beat it.

When he returned home, he was convinced that a winning system could be devised with the help of a high-speed electronic calculator. He used an IBM 704 computer to improve the strategy that he had used on his trip and called it "basic strategy".

His calculations showed that in a typical casino the house edge against the basic strategy is a mere 0.21 per cent.

After calculating basic strategy, he calculated the effect of removal. He found out that the removal of some cards had a positive effect on the player's advantage and the removal of others had a negative effect. Fives had the biggest effect, so Thorp decided to calculate a

strategy for counting fives. If a five was removed from the deck, it had a positive effect on the player's expectation, so when a deck was poor in fives, he would increase his bets. Also, the basic strategy would change a little according to how many fives were still in the deck. Removing four fives would give the player a 3.3 per cent edge.

Then it occurred to Thorp that although the effect of removal of tens wasn't as big as the effect of removal of fives, there were four times as much tens in a regular deck as fives. So the overall effect might be even bigger. This concluded into a winning strategy based on counting tens. Since the removal of a ten had a negative effect on the player's advantage, Thorp would calculate the ratio "others/tens" and whenever this ratio dropped below a certain number (2.00), the player would have the edge.

The advantages of the ten-count over the five-count are the following. The advantages for the ten-count strategy generally range from 1 to 10 per cent. The large advantages yield heavy winnings and the small advantages give the player camouflage: it turns out to be natural in this strategy to vary the bet size with the advantage in small steps. This is less conspicuous than making essentially two types of bets, "large" and "small".

After all this, he considered the effects of all the different cards when removed from the deck. He assigned a positive value to the cards that had a positive effect and a negative value to cards that had a negative effect. Of course, these values were all different, because the cards had different effects. Now you had to add these values as you saw the cards. This was called the running count. Whenever your running count is positive, it means that the total effect of removed "bad" cards is bigger than the total effect of the removed "good" cards, so you have the edge.

This all is rather difficult, so in the second edition of *Beat the Dealer*, the simple point count system was introduced. Julian Braun, who did a lot for the blackjack community calculating all kinds of important numbers, made most calculations for this second edition. In this system, the cards 2 through 6 get a value of +1, the 7, 8 and 9 get a value 0 and tens and aces get a value -1. This made keeping a running count much easier. There was one more point to be made however. When you have for example a running count of +3, you know that there are three more tens and aces than low cards still in the deck, hence you have an advantage. This advantage is bigger if there are fewer cards remaining. This idea resulted in the complete point-count system. This system is the same as the simple point-count system, except that you convert your running count into a true count. This means that you divide your running count by the number of decks that are still in play.

For the games in the 1960's it meant you had to divide by a number smaller than one, but as Thorp mentioned, this can easily be used if you ever encounter a game that uses more than one deck, which is common these days. It is this complete point-count system that is known today as the Hi-Lo system and that is used by a great number of card counters.

How It May End

Now we wonder if the profitable black jack games will ever stop existing. There are some measures the casinos could take to prevent counting. Firstly, there are the rule changes. After the publication of the first edition of *Beat the Dealer*, casinos made great changes in the rules. Fortunately, also the non-counting players started to complain and the rules were changed back. The nice thing is that Thorp foresaw these rule changes and wrote a chapter about how to handle these in the first edition of his book. In the second edition, he mentions that there were indeed made rule changes and that the casino personnel probably hadn't read further than

chapter five of his first edition. In this chapter the counting strategy was explained. The handling of the rule changes was in chapter eight.

The rule change that affects the counter the most, is the use of more decks in combination with terrible deck penetration, which is a term for the amount of cards that are dealt before the deck is shuffled. The favourable situations won't occur quite as often and they won't be as favourable as they used to be in a single deck game.

The second measure a casino can take is barring you from play. In the eighties, Ken Uston went to court and made sure that in New Jersey casinos can't bar you if you're counting. As a result, the casinos in Atlantic City have terrible rules. The rules in Nevada are better, but there you can still be barred from play if you're identified as a counter.

The third method of stopping counters is the CSM, the continuous shuffle machine. After every round of play the cards are put back in the machine and are shuffled back in the decks. Counting becomes useless. Unfortunately, these machines are all over the place in the Dutch casinos, so the only way I can help you improve your black jack play in this country is to give you the basic strategy for these casinos.

In America, there will probably be beatable games as long as there is blackjack, because of the great competition there is between casinos. So if you do want to become a card counter, you will have to look for good games abroad. Before you do that, I suggest you learn a little more about card counting and practice, since card counting isn't easy.

References

Books I can recommend are:

- Beat the Dealer, by Edward Thorp, because it's the book that started all this.
- The Theory of Black Jack, by Peter Griffin, if you are interested in the exact mathematics behind the game of blackjack.
- Professional Blackjack by Stanford Wong. This book has the most complete charts for all kinds of different rules.
- Blackjack Attack by Don Schlesinger in which there are covered a lot of different mathematical side aspects like risk of ruin, the illustrious 18 and SCORE.

For practicing counting, I suggest you go to

<http://blackjack-basics.com/bjd/bb.html>

Here you can practice counting with different count systems, but you can also practice using the indices for basic strategy departure.

Appendix

The Rules

Number of Players

A blackjack game has a dealer and from one to seven players.

The Pack

One ordinary 52-card pack is used. These days it is very difficult to find a single deck game. Most games use six or even eight decks.

The Deal

Before play begins, the cards are shuffled by the dealer and cut by the player. Next, a card is "burned" (placed face up on the bottom of the deck). This is because the top card might have been seen during the cut. The dealer then deals two cards to himself and to each of the players. Players get both cards face down and the dealer receives one card face up and one card face down. These days in some casinos the player's cards are dealt face up and the dealer receives his second card after all hands have been played.

Betting

The players place all bets except insurance before any cards are dealt. The house establishes a minimum bet and a maximum bet.

Numerical Value of the Cards

The player can choose either 1 or 11 as the value of an ace. The numerical value of a face card is 10, and the numerical value of the other cards is simply their face value.

Object of the Player

Each player tries to obtain a total that is greater than that of the dealer, but which does not exceed 21.

Naturals

If the first two cards dealt either to the player or to the dealer consist of an ace and a ten-value card, they constitute what we shall call a "natural" or "blackjack". If a player has a natural and the dealer does not, the player receives 1.5 times his original bet from the dealer. If a player does not have a natural and the dealer does, the player loses his original bet. If both dealer and player have naturals, no money changes hands.

The Draw

The draw starts at the left of the dealer and proceeds in clockwise fashion. A player looks at his hole cards and may elect to "stand" (draw no additional cards). Otherwise, he can request additional cards from the dealer, which are dealt face up, one at a time. After each player has drawn his cards, the dealer turns up his hole card. If his total is 16 or less, he must draw a card and continue to draw cards until his total is 17 or more.

The Settlement

If the player busts, he immediately loses his original bet. If the player does not go over 21 and the dealer does, the player wins an amount equal to his original bet. If neither player nor dealer busts, the person with the higher total wins an amount equal to the original bet of the player. If player and dealer have the same total, not exceeding 21, no money changes hands.

Splitting Pairs

If the player's hole cards are numerically identical, he may choose to split. For this, he turns his cards face up and uses them as the initial cards of two separate hands.

Doubling Down

After looking at his hole cards, the player may choose to "double down". He then doubles his bet, but receives one and only one more card.

Insurance

If the dealer's up card is an ace, an additional wager is allowed before the draw. A player may put up an additional side bet equal to half his original bet. After the player has decided whether or not to do this, the dealer checks his hole card. If the dealer has a natural, the side bet wins twice its amount. If the dealer does not have a natural, the side bet is lost. The original bet is settled in the usual way, regardless of the side bet.

Basic Strategy Charts

Given below is the basic strategy for the game of black jack as it is played in Holland Casino. The informed reader might notice that there are some differences from more well-known basic strategies. This is due to rule variations that are or are not applicable in the Holland Casino game. For example, most basic strategies tell the player to double down on a total of 11 against a dealers 10. This basic strategy doesn't, since the player loses all to a dealers black jack.

Pair Splitting

	2	3	4	5	6	7	8	9	10	A
2,2	S	S	S	S	S	S	-	-	-	-
3,3	S	S	S	S	S	S	-	-	-	-
4,4	-	-	-	S	S	-	-	-	-	-
5,5	-	-	-	-	-	-	-	-	-	-
6,6	S	S	S	S	S	-	-	-	-	-
7,7	S	S	S	S	S	S	-	-	-	-
8,8	S	S	S	S	S	S	S	S	-	-
9,9	S	S	S	S	S	-	S	S	-	-
10,10	-	-	-	-	-	-	-	-	-	-
A,A	S	S	S	S	S	S	S	S	S	-

S = Split

- = Don't Split

Doubling Down, Hitting and Standing on Hard Totals

	2	3	4	5	6	7	8	9	10	A
17 or higher	-	-	-	-	-	-	-	-	-	-
13-16	-	-	-	-	-	H	H	H	H	H
12	H	H	-	-	-	H	H	H	H	H
11	D	D	D	D	D	D	D	D	H	H
10	D	D	D	D	D	D	D	D	H	H
9	H	D	D	D	D	H	H	H	H	H

8 or lower	H	H	H	H	H	H	H	H	H	H
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D = Double Down (if doubling is not allowed then hit)

H = Hit

- = Stand

Hitting and Standing on Soft Totals

	2	3	4	5	6	7	8	9	10	A
19 or higher	-	-	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	H	H	H
17	H	H	H	H	H	H	H	H	H	H

H = Hit

- = Stand

Calculation of Basic Strategy

Although it is necessary to use a computer to calculate the exact basic strategy, I will give an outline of the mathematics that is used in these calculations. This outline can be found in more detail in Chapter 2 of *The Theory of Black Jack* by Peter Griffin.

As an example consider the choice of whether to draw or stand with (T,6) against a dealer 9. If the player stands on this total of 16, he will win or lose solely on the basis of whether the dealer busts. The dealer's chance of busting can be found by pursuing all of the 566 distinguishably different drawing sequences and weighting their paths according to their probability of occurrence. Once we have done this, we know that the dealer's chance of busting is 0.2304. This means we win 0.2304 bets for every 0.7696 bets we lose, resulting in a standing expectation of -0.5392. This means that we will lose approximately 54 cents on every dollar we bet if we stand on (T,6) against a dealer's 9.

Now we have to calculate our expectation if we draw one card. This means we have to calculate our conditional expectation for every possible card we can draw, given that we draw that card. If we draw a 2 for example, we have to calculate our expectation of our (T,6,2) against the dealer's 9. And we have to do this for every possible card we can draw. This is considerably more work than we needed for the standing expectation, since not only we have to calculate the probability of the dealer busting, but also the probabilities of the dealer ending up with a total of 17, 18, 19, 20 and 21. After we have done this, we can calculate our drawing expectation in the same way as before. It turns out that our drawing expectation is -0.4793. So we lose about 48 cents for every dollar bet if we draw to our (T,6) against the dealer's 9. Since a loss of 48 cents by drawing is preferable to one of 54 cents from standing, basic strategy is to draw to (T,6) against a dealer's 9.

Analysis of our best strategy with a smaller total of possibly more than two cards, such as (5,4,3), would be based on a sort of recursive reference to previous calculations of our optimal expectation and strategy with (5,4,3,A), (5,4,3,2),..., (5,4,3,9), (5,4,3,a,a), etc.